"COMPTON EFFECT"

Group C, paper IV, B.Sc. (Hons) part II
(Dr. Usha Kumari)

The fact that a photo-electron was emitted from the metal surface on the incidence of light without any time lag showed that the ejection of the electron was due to the impact of a particle. This suggest that the energy quantum ($h\nu$) has a momentum. The photon has no mass but it has a momentum and energy. That the photon can be treated as a massless particle with momentum and energy was fully demonstrated in Compton Effect involving the scattering of a photon by an eletron.

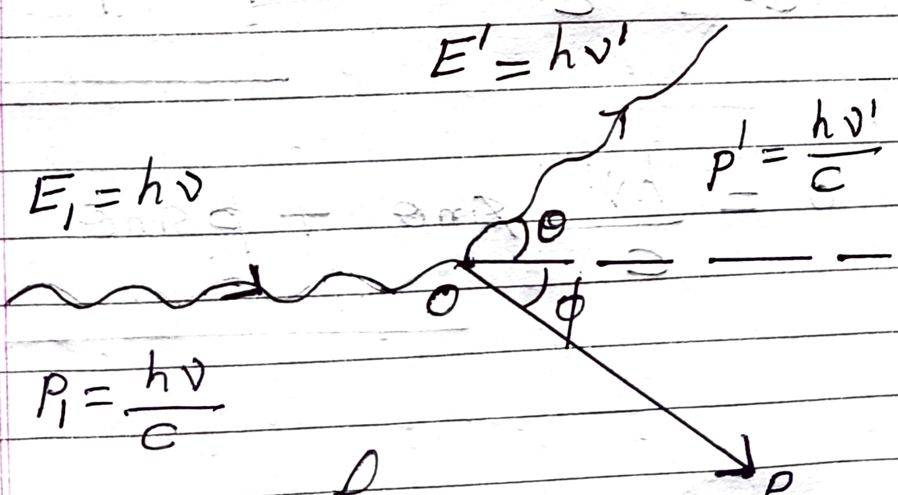

fig :- 1

A photon of mass zero, momentum $\frac{h\nu}{c}$ and energy $h\nu$ impinges on an electron of rest mass m_0 at O.
The photon is scattered with reduced energy $h\nu'$ and momentum $\frac{h\nu'}{c}$ in a direction making an angle θ with the initial direction.
The electron recoils in a direction ϕ with momentum p.
and
total energy $= \sqrt{p^2c^2 + m_0^2c^4}$
Momentum is conserved and mass-energy is conserved.
Conservation of momentum yields

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\phi \quad \text{——— (1)}$$

$$0 = \frac{h\nu'}{c}\sin\theta + p\sin\theta \quad \text{——— (2)}$$

Conservation of energy gives

$$h\nu + m_0 c^2 = h\nu' + \sqrt{p^2c^2 + m_0^2c^4} \quad (3)$$

$$\Rightarrow h(\nu - \nu') + m_0 c^2 = \sqrt{p^2c^2 + m_0^2 c^4}$$

Squaring on both sides

$$h^2(\nu-\nu')^2 + m_0^2 c^4 + 2m_0 c^2 h(\nu-\nu') = p^2c^2 + m_0^2 c^4$$

$$\Rightarrow h^2(\nu-\nu')^2 + 2m_0c^2h(\nu-\nu') = p^2c^2 \quad (4)$$

Since from equn (1) & (2)

$$p\cos\phi = \frac{h}{c}(\nu - \nu'\cos\theta)$$

$$p\sin\phi = -\frac{h\nu'}{c}\sin\theta$$

$$\therefore p^2 = \left(\frac{h}{c}\right)^2 \left\{(\nu-\nu'\cos\theta)^2 + \nu'^2\sin^2\theta\right\}$$

$$= \left(\frac{h}{c}\right)^2 \left\{\nu^2 + \nu'^2\cos^2\theta - 2\nu\nu'\cos\theta + \nu'^2\sin^2\theta\right\}$$

$$= \left(\frac{h}{c}\right)^2 \left\{\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right\}$$

$$p^2c^2 = h^2\left\{\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right\} \quad (5)$$

Equating values of p^2c^2 from eqn (4) & eqn (5) we have :-

$$h^2\{\nu^2+\nu'^2-2\nu\nu'\cos\theta\} = h^2\{\nu^2+\nu'^2-2\nu\nu'\} + 2m_0c^2h(\nu-\nu')$$

$$\Rightarrow -2h^2\nu\nu'\cos\theta = -2\nu\nu'h^2 + 2m_0c^2h(\nu-\nu')$$

$$\Rightarrow h\nu\nu'(1-\cos\theta) = m_0c^2(\nu-\nu')$$

$$\Rightarrow \frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\Rightarrow \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\Rightarrow \frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\theta)$$

$$\boxed{\therefore \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

Where

$\frac{h}{m_0c}$ is called the

Compton Wavelength.

—x— The end —